Component Mode Synthesis Methods for Test-Based, Rigidly Connected Flexible Components

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With the objective of enhancing test-based modal synthesis and planning the development of test methods, simple examples were run using finite element-based components to compare accuracy and define test requirements for the following approaches to component mode synthesis: restrained modal component, residual flexibility, mass-loaded connect degrees of freedom, and rotational connect degrees of freedom.

Nomenclature

F = applied force
G = flexibility
K, k = physical and modal stiffness, respectively
M, m = physical and modal mass, respectively
N = numbers of retained modes
q = physical degrees of freedom
R = reaction force
T = geometric transformation
Z = residual flexibility
η = modal degree of freedom
Ψ = mode shape
ω = frequency

Subscripts

b = degrees of freedom free during component analysis, to be connected in system analysis
c = connect degrees of freedom
f = redundant connect degrees of freedom
i,j = physical connect degrees of freedom
η = modal degree of freedom

I. Introduction

RECENT advances in modal testing methods and software[1-7] have reduced the time and increased the accuracy and confidence associated with experimental determination of modal properties. This is particularly true for structures formerly considered very difficult to test, such as lightweight flight components with closely spaced modes or rocket motors that have high damping. These new modal testing techniques motivate consideration of new uses for test methods in structural design, analysis, and validation. Typically for aerospace structures, modal testing is used only to verify analytical modes. The work reported here was initiated to increase the feasibility of using modal testing methods in two areas: 1) experimental component structural characterization as an alternative to analysis in conceptual or early design activities, and 2) component testing and component mode synthesis as an alternative to large-scale total space vehicle testing.

For design purposes, components of dynamic systems are often analytically modeled in order to obtain sufficient understanding to select a design concept. For some components, the analysis requires substantial time and cost commitment or unacceptable uncertainty because of the complexity of the structure. With recent new capabilities, component modal testing instead of analysis may be the most effective way to obtain a simple simulation of the system dynamics. Although the actual article being designed does not yet exist, representative components or subunits of the structure can sometimes be fabricated. As long as confidence exists in the component mode synthesis techniques, these test-based component characteristics can be assembled into a simulation of new design concepts. The achievement of design models from test is one of the objectives of this study.

The other objective is to meet the demands of verifying the structural integrity of very large dynamic systems. As space vehicles get larger, the feasibility of testing the complete assembled system is lost due to the size of currently available test facilities. If confidence exists in test-based modal synthesis tchniques, only the components of the vehicle need to be tested. Final system performance can be verified through analysis using component mode synthesis.

The structural definition of components totally from test has been accomplished and used successfully for design purposes[8,9] with relatively stiff structures connected with flexible elements such as an automobile frame and body connected with isolation mounts. For this situation, components can be tested with free boundaries to obtain a free-free modal data base. These modal data are sufficient to use in component mode synthesis to simulate system dynamic performance. However, in the case of aerospace vehicles, the components are typically more flexible and are connected more rigidly. For large space structures, greater extremes in flexibility of rigidly connected components are expected. In general, using a truncated set of free-free modes to simulate aerospace systems leads to intolerable errors in the system response predictions.

Many methods for obtaining a more accurate component definition have been developed for component analytical modeling. The exercise described here was undertaken to help in planning the development of improved experimental methods. The main interest is in selecting what additional measurement methods should be developed to obtain accurate definition for flexible components to be rigidly connected.

II. Approach

The approach to evaluating the importance of various additional or different modal test measurements was to solve a simple system, representative of the actual system of interest, with different component mode synthesis methods. For this study, modal analysis was carried out by finite element analysis.

In each case, the system synthesis was carried out with the Structural Dynamics Research Corporation (SDRC) SYSTAN[10] dynamic analysis program. The component finite element analysis was performed using either SDRC SUPERB[11] or MODEL SOLUTION[12] or MSC NASTRAN[13] programs, all of which are interfaced with SYSTAN as part of the SDRC I-DEAS™ [14] software package. The accuracy of

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the various methods was assessed by comparing the frequencies, mode shapes, and strain energy distribution to a finite element solution of the whole system with both components included.

The structural configuration selected for this study is a simple representation of a flexible satellite supported in a relatively rigid cradle structure. The cradle structure was represented by beams and the satellite by parabolic shells. This structure and simplified model are shown in Fig. 1.

The various component synthesis methods studied were applied to the satellite component definition while the cradle structure was always represented by the same modal data base, which consisted of 20 modes with degrees of freedom to be connected left free during the modal analysis.

The results from the total finite element model of the system in terms of frequencies and strain energy distribution are given in Table 1. These results were treated as the reference for evaluating the accuracy of the other methods. The strain energy distributions for each mode were especially valuable for making comparisons particularly related to the accuracy of the load determination for a component. The example problem was selected to have a significant amount of strain energy in the satellite component so that changes in the methods for definition of this component would show an effect in the system response predictions.

The first form of component mode synthesis tried was to perform free-free modal analysis on the satellite defined by just 10 modes. Within the SYSTAN program, a system was formed with these 10 satellite modes connected with very stiff translational springs to the 20 modes of the cradle. The results are summarized in terms of frequencies and strain energy distribution in Table 2.

The results in Table 2, when compared with Table 1, illustrate the difficulties that occur when free-free testing and a severely truncated set of modes are used to define a component. Although some modes are reasonably close in frequency, the distribution in strain energy is not at all accurate and would lead to a very poor estimate of satellite loads. The total finite element model results of Table 1 show that, in modes 2 and 3 in particular, most of the strain energy should be in the satellite; yet the free-free component definition leads to zero strain energy in the satellite in these important modes.

Extending the satellite model to include 20 free-free modes led to the results in Table 3. Doubling the number of satellite component modes leads to an improvement in system response predictions. However, the simulation still does not provide a close approximation of the total finite element result or a good representation of loads in the satellite.

Several methods for improving component mode synthesis over these results for a free-free modal satellite component were tried on this mode. These candidate methods are summarized next.

III. Summary of Candidate Methods for Accuracy Improvement

In order to avoid using an unreasonable number of free-free modes, other synthesis methods are often used. The literature

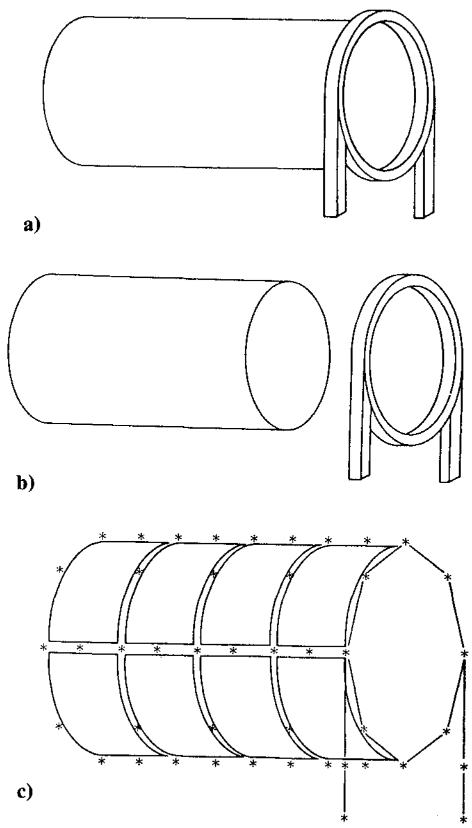

Fig. 1 Satellite-cradle systems used as example to compare methods. a) example: system; b) example: components; c) finite element models.

Table 1 Frequency and strain energy results for the first 5 system modes of the total finite element model

	Modes				
	1	2	3	4	5
Frequency, Hz	1.16	1.63	1.71	2.15	3.00
Satellite strain energy, %	19	67	74	25	29
Cradle strain energy, %	81	33	26	75	71
Connection strain energy, %	–	–	–	–	–

Table 2 Frequency and strain energy results for first 5 system modes using a free-free satellite modal component with 10 modes and only translational degree-of-freedom connnections

	Modes				
	1	2	3	4	5
Frequency, Hz	1.27	2.50	3.29	3.94	6.11
Satellite strain energy, %	–	–	3	–	–
Cradle strain energy, %	99	99	100	97	100
Connection strain energy, %	1	–	–	–	–

Table 3 Frequency and strain energy results for first 5 system modes using a free-free satellite modal component with 20 modes and only translational degree-of-freedom connections

	Modes				
	1	2	3	4	5
Frequency, Hz	1.27	2.22	2.31	2.49	3.56
Satellite strain energy, %	1	36	10	8	4
Cradle strain energy, %	99	64	90	92	96
Connection strain energy, %	–	–	–	–	–

includes many different methods of component mode synthesis[15-23] as well as reviews comparing methods.[23-30] Although a single review may recommend a particular method, which appears the most accurate, there is not a clear consensus of one method over all others. The accuracy of a particular method depends in part on the problem used to test it. Moreover, the practical difficulty (cost, time, equipment, software, experience) of acquiring the information needed to carry out each method varies. Therefore, selecting a method for test-based modal modes still requires sample problems to assess the relative practicability, accuracy, and overall acceptability of the candidate methods.

For convenience in this study, the various methods were grouped into four general types defined below. The first two groups of methods were considered because they have been described in the literature as the most accurate, particularly for rigid connection of flexible components.

Restrained Modal Component

The methods grouped here use modal data from a test or analysis with the connect degrees of freedom fixed; that is, those degrees of freedom that will be used to connect a component to other components when the system is assembled have a boundary condition of zero displacements. The methods of Hurty[20] and Craig and Bampton[16] are the best-known examples of this approach. In general, these methods are expected to be most accurate when the final system allows the component of interest to have little motion near the connect degrees of freedom. Another way to describe this situation is that a flexible component is rigidly attached to a relatively stiff component. The mass associated with the connect degrees of freedom is often neglected, while the local stiffness is accurately included with methods that take this general approach.

Free-Free Modal Component with Residuals

These methods, due to MacNeal[22] and Rubin,[23] improve the truncated free-free modal representation of a component by including estimates of the residual effects due to modes higher in frequency than the cutoff frequency for including modes in the component representation. In general, the residual effects are obtained by calculating the component flexibility due to those modes to be retained and then subtracting this from the total flexibility of the component, which is obtained either from a finite element stiffness matrix or an experimentally determined frequency response function for the component. Once this "residual" flexibility as been obtained, it can be used to enhance the component modal stiffness representation before the component is connected into the system. In the Rubin approach, the first-order response based on the residual stiffness is used further to estimate the residual inertial and damping effects of higher-order modes.

For the sample problems reported in the literature,[23] the inclusion of residuals appears as accurate as, or even more accurate than, restrained modal approaches. For components from test, the residual flexibility approach is particularly appealing because the free-free modal testing can be used. Free-free modal testing with the inclusion of residual flexibility has been proved feasible.[8,9]

The two remaining approaches were attempted on the basis of potential test feasibility rather than known accuracy improvements.

Mass-Loaded Connect Degrees of Freedom

The limitations of any free-free modal approach in defining the local flexibility at connect locations is the lack of deformation in these areas in the lower-frequency modes. It has been suggested that mass-loading these bounday degrees of freedom during the modal analysis would force measurable deformation in this critical area in the first few component modes. By removing the mass (adding negative mass) in the system synthesis, the true system could be simulated—perhaps with more accuracy for a given number of modes than could be obtained with the unaugmented component free-free modes.

Rotational Connect Degrees of Freedom

Another approach at better definition of local flexibility at connect locations is to measure rotational degrees of freedom as well as translations during the modal test. These additional degrees of freedom could potentially include enough added information to allow a more accurate component definition.

Rotational degrees of freedom are not normally included in the modal test because they are not particularly helpful in plotting and visualizing mode shapes and because translational accelerometers are the usual transducer used to record motion in a modal test. However, various indirect methods for measuring rotations have been developed.[31,32] In addition, a practical rotation accelerometer may be available soon, if not already.

IV. Results for Candidate Approaches Applied to Satellite-Cradle Example

Results for each of the candidate approaches are summarized here for the satellite-cradle system shown in Fig. 1. Also included here is an explanation of each method and a discussion of data that must be measured for experimental definition of a component via each method.

Restrained Modal Component

In order to determine the accuracy and feasibility of simulating the satellite component with a restrained modal approach, a modal analysis of the satellite was performed using SDRC SUPERB with zero deflections boundary conditions at connect degrees of freedom. These modes were then assembled using the Hurty method to define the satellite component, which was then connected to the same cradle component used before. (In this system, no connectors are required because the connect degrees of freedom are not constrained by the mode shapes.) The results are summarized in Table 4. Comparison of Table 4 and Table 1 shows that the constrained mode approach provides very accurate results. The satellite strain energy distribution matches that of the total finite element case.

In order to understand requirements for a restrained modal test for component mode synthesis, the elements required to formulate the component stiffness and mass matrices for system assembly are reviewed here.

A general schematic diagram of the type of system being considered is shown in Fig. 2. For a component A, the mass and stiffness matrices $[K]$ and $[M]$ can be partitioned into the following sets: $\{q_m\}$ = connection degrees of freedom which were fixed during the subsystem analysis; $\{q_r\}$ = remaining degrees of freedom in component A.

The equations of motion for the component A can then be written as

$$\left[-\omega^2\left(\begin{array}{c|c}M_{rr} & M_{rm}\\ \hline M_{rm}^T & M_{mm}\end{array}\right)+\left(\begin{array}{c|c}K_{rr} & K_{rm}\\ \hline K_{rm}^T & K_{mm}\end{array}\right)\right]\left\{\begin{array}{c}q_r\\ \hline q_m\end{array}\right\}=\left\{\begin{array}{c}F_r\\ \hline F_m\end{array}\right\} \quad (1)$$

Assume that $[M_{rm}] \simeq [M_{mm}] \simeq 0$. Inherent in the assumption that system modes are more like restrained modes than free-free modes is the assumption that the degrees of freedom $\{q_m\}$ will not move very much in the system; therefore, their mass is neglected. Rewriting the component A equation (1) with zero mass at connect degrees of freedom results in the following equation:

$$\left[\begin{array}{c|c}[K_{rr}-\omega^2 M_{rr}] & [K_{rm}]\\ \hline [K_{rm}^T] & [K_{mm}]\end{array}\right]\left\{\begin{array}{c}q_r\\ \hline q_m\end{array}\right\}=\left\{\begin{array}{c}F_r\\ \hline F_m\end{array}\right\} \quad (2)$$

Table 4 Frequency and strain energy results for first 5 system modes using a restrained modal component with 10 satellite modes and only translational degree-of-freedom connections

	Modes				
	1	2	3	4	5
Frequency, Hz	1.17	1.64	1.71	2.18	3.09
Satellite strain energy, %	20	68	74	26	29
Cradle strain energy, %	80	32	26	74	71
Connection strain energy, %	–	–	–	–	–

Component A is tested with $\{q_m\}=0$. The result is a matrix of mode shape vectors, $[\Psi]$ for which

$$[\Psi]^T[K_{rr}][\Psi]=[\lceil k \rfloor], \qquad [\Psi]^T[M_{rr}][\Psi]=[\lceil m \rfloor]$$

Let $\{n\}$ represent the modal coordinates and $\{F_\eta\}$ the modal forces such that

$$\{q_r\}=[\Psi]\{\eta\}, \qquad \{F_\eta\}=[\Psi]^T\{F_r\}$$

Multiply the upper partition of Eq. (2) by $[\Psi]^T$ and use the above definitions to obtain the following component definition equation:

$$\begin{bmatrix} [k]-\omega^2 & [m] & [\Psi]^T[K_{rm}] \\ [K_{rm}]^T & [\Psi] & [K_{mm}] \end{bmatrix} \begin{Bmatrix} \{\eta\} \\ \{q_m\} \end{Bmatrix} \begin{Bmatrix} \{F_\eta\} \\ \{F_m\} \end{Bmatrix} \tag{3}$$

The modal stiffnesses $[\lceil k \rfloor]$, modal masses $[\lceil m \rfloor]$, and mode shape vectors $[\Psi]$ are readily obtained from the restrained modal test or analysis, but the stiffnesses $[K_{mm}]$ and $[K_{rm}]$ associated with the restrained connect degress of freedom are not directly defined by a modal test or analysis. These additional stiffnesses must be obtained from measurements of modal reaction forces in the restrained modal test and from static measurements of stiffnesses between connect points. This process of obtaining these additional stiffnesses is explained further in the following paragraphs.

During the modal test, $\{q_m\}=0$; from Eq. (3),

$$[K_{rm}]^T[\Psi]\{\eta\}=\{F_m\} \tag{4}$$

The matrix $[K_{rm}]^T\ [\Psi]$ is a matrix of reaction forces associated with each mode $[R]$.

The matrix of modal reaction forces can be measured directly from modal test by using a force transducer at the connect locations during the modal test and extracting modal forces as part of the eigenvectors during modal analysis. Thus Eq. (3) becomes

$$\begin{bmatrix} [\lceil k \rfloor]-\omega^2[\lceil m \rfloor] & [R]^T \\ [R] & \{K_{mm}] \end{bmatrix} \begin{Bmatrix} \{\eta\} \\ \{q_m\} \end{Bmatrix} = \begin{Bmatrix} \{F_\eta\} \\ \{F_m\} \end{Bmatrix} \tag{5}$$

in which the only unknown for definition of component A is the matrix $[K_{mm}]$.

In order to evaluate $[K_{mm}]$ in terms of known or measurable quantities, rearrange Eq. (5) into the following form:

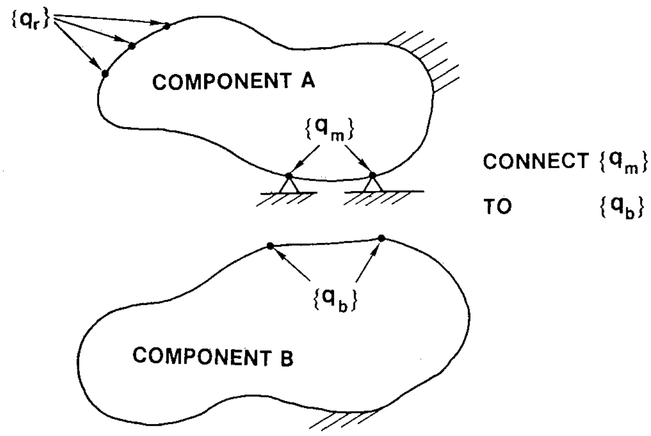

Fig. 2 General schematic of a system with a constrained modal component.

The term

$$\{\{F_\eta\}+\omega^2[\lceil m \rfloor]\{\eta\}\}=\{F'\}$$

represents the forces on the free degrees of freedom. For the static case, $\{F'\}$ is equal to zero. Therefore, from Eq. 6, the following term is the static stiffness among connect degrees of freedom:

$$[K'_{mm}]=[K_{mm}]-[R][\ k\ ]^{-1}[R]^T \tag{7}$$

If the component restraints are statically nonredundant—that is, the restraints are the minimum required to prevent rigid body motion—then there is zero stiffness among the restraints and

$$[K'_{mm}]=0$$

and

$$[K_{mm}]=[R][\lceil k \rfloor]^{-1}[R]^T \tag{8}$$

In this special case the $[K_{mm}]$ matrix can be formed from the modal reaction forces and modal stiffness values alone.

For most structures, the restraints $\{q_m\}$ are redundant, and $[K'_{mm}]$ does not equal zero and must be evaluated. In order to make this evaluation, consider partitions of $\{q_m\}$ into nonredundant subset $\{q_c\}$ and other degrees of freedom $\{q_f\}$.

$$\{q_m\}=\begin{Bmatrix} \{q_f\} \\ \{q_c\} \end{Bmatrix}$$

The evaluation of $[K'_{mm}]$ requires an additional static analysis or test for which only a nonredundant set of degrees of freedom $\{q_c\}$ is restrained. With three restraints, the static deflection due to some applied force $\{F_f\}$ of the other $\{q_f\}$ degrees of freedom can be expressed as the sum of rigid body motion and elastic deformation of the component:

$$\{q_f\}=\underbrace{\overset{\text{flexibility matrix}}{[G]}\{F_f\}}_{\text{elastic deformation}}+\underbrace{\overset{\text{geometric transformation}}{[T]}\{q_c\}}_{\text{rigid body motion}} \tag{9}$$

$$\left[\begin{array}{c|c} [\lceil k \rfloor]^{-1} & -[\lceil k \rfloor]^{-1}[R]^T \\ \hline [R][\lceil k \rfloor] & [K_{mm}]-[R][\lceil k \rfloor]^{-1}[R]^T \end{array}\right] \left\{\begin{array}{c} \{F_\eta\}+\omega^2[\lceil m \rfloor]\{\eta\} \\ \hline \{q_m\} \end{array}\right\} = \begin{Bmatrix} \{\eta\} \\ \{F_m\} \end{Bmatrix} \tag{6}$$

Also from static equilibrium, the forces applied at $\{q_f\}$ must be balanced by forces at $\{F_c\}$ and at $\{q_c\}$; therefore,

$$-[T]^T\{F_f\} = \{F_c\} \tag{10}$$

Rearranging Eqs. (9) and (10) and writing as a matrix results in the following equations:

$$\underbrace{\begin{bmatrix} [G]^{-1} & -[G]^{-1}[T] \\ -[T]^T[G]^{-1} & -[T]^T[G]^{-1}[T] \end{bmatrix}}_{[K'_{mm}]} \begin{matrix} \{q_f\} \\ \{q_c\} \end{matrix} = \begin{matrix} \{F_f\} \\ \{F_c\} \end{matrix} \tag{11}$$

Therefore, the total $[K_{mm}]$ matrix has been evaluated—and thereby the total component A definition matrix.

What must be measured in a separate static test is the stiffness matrix $[G]^{-1}$. Therefore, the following items must be measured in the modal test: $[\diagdown m \diagdown]$, $[\diagdown k \diagdown]$, $[\Psi]$, $[R]$, $[G]^{-1}$, $[T]$. The first three items are just the modal mass, stiffness, and mode shape vectors that are obtained from a modal test with the component clamped. The next item $[R]$ is the matrix of modal reaction forces that can easily be extracted as part of the eigenvectors but requires transducers to record the forces required to restrain the structure during the modal test. The matrix $[G]^{-1}$ requires a static test for which a nonredundant subset of the connect degrees of freedom is constrained and the stiffness matrix of the redundant set is measured. The matrix $[T]$ is just a rigid body transformation between connect degrees of freedom that can easily be obtained from the geometry alone. Therefore, the new challenges for test are the measurement of restraint forces and stiffness matrix $[G]^{-1}$.

Residual Flexibility

If all modes are retained in a component definition, the modal representation will be exact. Therefore, the accuracy of the truncated mode representation can be improved by estimating the residual effects of the higher-order modes. Because residual flexibility can easily be obtained experimentally to supplement the free-free component definition and because the first-order residual flexibility correction is readily available in SYSTAN, NASTRAN, and MODAL-PLUS,[33] this approach was tried next on the same sample problem used above with the results shown in Table 5. Although this set of natural frequencies shows closer agreement with the full finite element result than use of the free-free modes alone, the accuracy in satellite strain energy distribution is still poor, particularly in comparison to the restrained mode approach.

In order to better understand the information used for this example, the calculation of residual flexibility is reviewed here. The dynamic flexibility or frequency response function G_{ij} can be expressed in terms of the modal stiffness k_r; mass m_r; and mode shape vectors Ψ_r by the following summation over the modes where the subscript r refers to the rth mode and the subscripts i and j refer to physical degrees of freedom, which in this case will be connect degrees of freedom.

$$G_{ij} = \sum_{r=1}^{N} \frac{\Psi_{ri}\Psi_{rj}}{k_r - \omega^2 m_r} + \sum_{r=N+1}^{\infty} \frac{\Psi_{ri}\Psi_{rj}}{k_r - \omega^2 m_r} \tag{12}$$

In this expression, N is the number of retained modes; the second term above is called the residual flexibility Z_{ij}, due to modes truncated in a simulation. Experimentally the term Z_{ij} can be obtained by exciting the structure at point i, measuring the response at point j, and subtracting the first sum in Eq. (12), which can be evaluated after the modal analysis has determined the properties of the first N modes. Using finite element analysis, Z_{ij} is approximated assuming that truncated modes have only a stiffness contribution. The static stiffness contribution of the retained modes is then subtracted from the physical static stiffness matrix to obtain a first-order or static approximation to Z_{ij}. Once an estimate of the residual effects at connect degrees is obtained, it is necessary to include these in the component stiffness and mass matrices.

Table 5 Frequency and strain energy results for 10 free-free satellite modes using residual flexibility

	Modes				
	1	2	3	4	5
Frequency, Hz	1.16	2.15	2.70	3.00	4.82
Satellite strain energy, %	19	25	19	31	30
Cradle strain energy, %	81	75	81	69	70
Connection strain energy, %	–	–	–	–	–

From the free-free modal test, the stiffness and mass matrices have only modal degrees of freedom. In order to add the residual effects at the connect degrees of freedom and to expand the stiffness matrix to include the connect degrees of freedom, the definition of residual effects is rewritten into the following stiffness matrix form.

$$\underset{\substack{\text{residual}\\\text{effects stiffness}}}{[\quad K_{\text{resid}} \quad]} \begin{Bmatrix} \{\eta\} \\ \{q_c\} \end{Bmatrix} = \begin{Bmatrix} \{F_\eta\} \\ \{F_c\} \end{Bmatrix} \tag{13}$$

where $\{\eta\}$ are the modal degrees of freedom, $\{q_c\}$ the physical connect degrees of freedom, and $\{F_c\}$ the forces applied to the connect degrees of freedom.

The physical displacements $\{q_c\}$ are given by the following equation

$$\{q_c\} = [\Psi]\{\eta\} + [Z]\{F_c\} \tag{14}$$

which can be rearranged to obtain

$$\{F_c\} = [Z]^{-1}\{q_c\} - [Z]^{-1}[\Psi]\{\eta\} \tag{15}$$

Equation (15) is one of the equations needed to fill out the K_{resid} matrix, and the other can be obtained by multiplying Eq. (15) by $-[\Psi]^T$ to obtain an expression for F_η. The resulting residual effects stiffness matrix formulation is the following:

$$\left[\begin{array}{c|c} [\Psi]^T[Z]^{-1}[\Psi] & -[\Psi]^T[Z]^{-1} \\ \hline -[Z]^{-1}[\Psi] & [Z]^{-1} \end{array}\right] \left\{\begin{array}{c} \{\eta\} \\ \hline \{q_c\} \end{array}\right\} = \left\{\begin{array}{c} \{F_n\} \\ \hline \{F_c\} \end{array}\right\} \tag{16}$$

The whole stiffness and mass matrix for the component can be obtained by adding the free-free modal mass and stiffness from the modal test into the upper partition of Eq. (16). When this formulation is included in the system synthesis, the connect degrees of freedom are now independent in the system formulation, and no auxiliary connectors are required.

We have not yet implemented the second-order correction suggested by Rubin.[23] Including the estimate of inertial and dissipative effects of the truncated modes should further improve the accuracy of the residual flexibility approach.

Mass-Loaded Connect Degrees of Freedom

Adding mass to exercise the local structure near connect locations was studied next as a potential experimental approach. This method consisted of attaching lumped masses at connect degrees of freedom during the modal analysis. Then, in the system synthesis, this modal component was attached with a connector that had equivalent negative mass at the connect locations. Cases were tried with 10 satellite modes run with the lumped mass at each of the eight connect locations equal to 0.01, 0.1, and 1.0 times the mass of the entire satellite. No improvements in accuracy over the free-free case of Table 2 were obtained. The most accurate results are shown in Table 6. These results are for each lumped mass equal to 1/10 the entire satellite mass. The strain energy observed in the connectors indicates a modal insufficiency problem: the condition that satellite connect degrees of freedom and corresponding cradle connect degrees of freedom have zero relative motion is not being satisfied. Several cases were run with increased numbers of modes. At 20 modes, there is no strain energy in the connectors but still no improvement over the 20 mode free-free case of Table 3. Even at 40 modes, the results are still not as accurate as 10 modes with residual flexibility.

Rotational Degrees of Freedom

The fourth approach tried was the inclusion of rotational degees of freedom. Again using the above models and 10 free-free modes of the satellite, the system synthesis included stiff translational and rotational springs connecting the satellite and cradle degrees of freedom. For this case, the results are summarized in Table 7. The results again show large deviations from the total finite element result. Note the strain energy in the connection indicating insufficient modes to satisfy the connection equations. The same example was run with 20 satellite modes and then with 40 satellite modes. Even for 40 modes, the connection equations are not satisfied and the results are not at all accurate.

This result was at first surprising because the addition of more degrees of freedom suggests a more accurate definition of a component. Because the example here showed a reduced accuracy with the addition of rotations, two other independent analyses of this same general configuration were run.

Table 6 Frequency and strain energy results for first 5 system modes using a free-free satellite modal component with 10 modes; mass = 0.1 × the mass of the satellite aws added used at connect degrees of freedom in component analysis and removed in system analysis

	Modes				
	1	2	3	4	5
Frequency, Hz	1.27	2.51	3.29	4.12	6.31
Satellite strain energy, %	–	–	–	–	–
Cradle strain energy, %	99	100	100	99	100
Connection strain energy, %	1	–	–	1	–

Table 7 Frequency and strain energy results for first 5 system modes using a free-free modal satellite component with 10 modes and translational and rotational degree-of-freedom connected in system analysis

	Modes				
	1	2	3	4	5
Frequency Hz,	4.17	7.78	15.12	19.1	32.2
Satellite strain energy, %	–	–	–	–	–
Cradle strain energy, %	99	68	35	81	1
Connection strain energy, %	1	32	65	19	99

One involved a cradle component of 1/20 the stiffness (1/20 the bending moment in the beams), and the other had this same flexible cradle but double the nodes and elements in both the cradle and satellite components. In both additional cases, a decrease in accuracy or an increased difficulty in satisfying the connecting condition was obtained with the addition of rotational degrees of freedom.

Although it seems counterintuitive that increasing the number of degrees of freedom can reduce the accuracy, the effect observed could be decribed instead as increased difficulty in the synthesis rather than reduced accuracy. The strain energy in the connectors indicates that the synthesis equations, which call for degrees of freedom on the satellite to be fixed to corresponding ones on the cradle, are not satisfied and, therefore, the synthesis has not been accomplished. With the addition of rotational degrees of freedom, more modes are required to approximate the synthesis constraint equations before a valid comparison in accuracy can be obtained.

V. Conclusions

The main objective of this current effort was to determine what measurements were required to define accurately, by experimental modal analysis, a flexible component for mode synthesis of a system with rigid connections and a configuration similar to the satellite-cradle example. In this example, the satellite is flexible enough to allow significant strain energy in this component.

The conclusion based on the work to date is that no significant improvement over unaugmented free-free modes can be expected from the inclusion of either rotational degrees of freedom or mass-loaded boundary degrees of freedom. Too many modes are required for either of these approaches to allow even a valid system synthesis. The use of residual flexibility with free-free modes is much more promising and is the only method that is immediately feasible experimentally. However, achieving acceptable accuracy for rigidly connected flexible components requires more modes than the restrained modal component approach does.

For the configuration tried here, the restrained modal component approach was significantly more accurate than even the residual flexibility approach. Therefore, the implementation of a second-order correction in residual effects (Rubin's approach) and consideration of the feasibility of measurement of restrained modes should be investigated further.

Antoher conclusion apparent from the examples reported in the literature and from the execution of the examples in this paper is that the relative accuracy of a method depends greatly on the nature of the example. Certainly the relative flexibility of the components in the system determines the importance of the local flexibility and local mass at the connect degrees of freedom. This conclusion suggests that it is worthwhile to make a very simple conceptual model that is representative of the system in question, yet small enough for a total finite element model check. Once various methods have been tried on the small model, the most accurate feasible method can be readily identified.

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